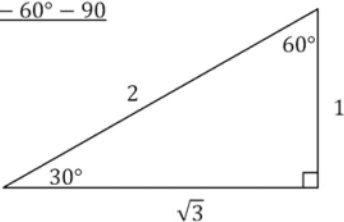
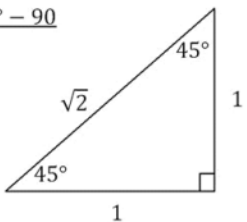


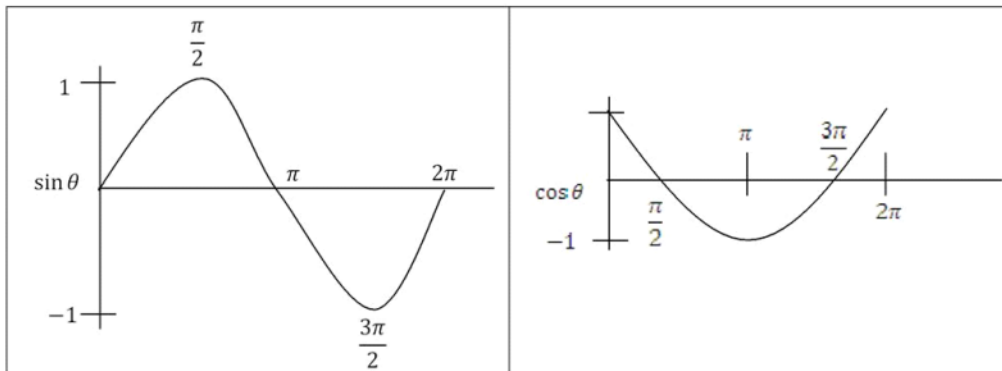
ACT COMPASS Review Packet Contents: College Algebra and Trigonometry

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Formulas

Trigonometry

$\sin \theta = \frac{opp}{hyp}$	$\cos \theta = \frac{adj}{hyp}$	$\tan \theta = \frac{opp}{adj} = \frac{\sin \theta}{\cos \theta}$
$\csc \theta = \frac{1}{\sin \theta}$	$\sec \theta = \frac{1}{\cos \theta}$	$\cot \theta = \frac{1}{\tan \theta}$
$\sin^2 \theta + \cos^2 \theta = 1$	$1 + \tan^2 \theta = \sec^2 \theta$	$1 + \cot^2 \theta = \csc^2 \theta$
<p><u>30° - 60° - 90°</u></p> 	<p><u>45° - 45° - 90°</u></p> 	



- For $\sin a\theta$ or $\cos a\theta$, period = $\frac{2\pi}{a}$ therefore, the smaller a is, the longer the period.
- **All Students Take Calculus** – tells which functions are positive in which quadrants. See figure below.

S	A
T	C

Complex Numbers

- Any four i terms with consecutive exponents = 0.
- $i^0 = 1$
 $i^1 = i$
 $i^2 = -1$
 $i^3 = -i$
- From this point on, the pattern repeats.
- $i^4 = 1$
 $i^5 = i$
 $i^6 = -1$
 $i^7 = -i$

Sample Problems

Functions

1. If $f(x) = x^2 - 3x + 2$ and $g(x) = 8x - 3$, then find $f[g(2)]$.
2. If $f(x) = x^2 - 3x + 5$, then find $f(a + 2)$.
3. If $f(x) = 8x + 5$ and $g(x) = x^2 - 7$, then find $2f(3) - 4g(-2)$.

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4. If $d = 1000 - 200q$ and $c = 4000 + 20d$, then find c as a function of q .
5. If $f(x) = x^2 - 3x + 2$ and $g(x) = 5 - x$, then find $f[g(x)]$.
6. If $f(x) = 2x^2 + 1$ and $g(x) = 6x^3 - 2x^2 + 3x - 1$, then find $\frac{g(x)}{f(x)}$.
7. If the point $(5, -7)$ is on the graph of $y = f(x)$, what point must be on the graph of $y = f^{-1}(x)$?
8. What is the domain of $f(x) = \sqrt{x^2 - x - 6}$?
9. If $f(x) = \frac{2x+5}{3}$, then find $f^{-1}(x)$.

Exponents

10. If $4^x = 32$, then find x .
11. Simplify $27^{-\frac{2}{3}}$
12. If $(\frac{2}{5})^{2x+4} = (\frac{5}{2})^{6x-5}$, then find x .
13. If $(a^{x^2})\left(\frac{1}{a^{8x}}\right) = \frac{1}{a^{16}}$, then solve for x .
14. If $(2^{x^2})\left(\frac{1}{2^{4x}}\right) = \frac{1}{16}$, then solve for x .

Complex Numbers

15. Multiply: $7i(4 - 3i)$
16. Multiply: $(4 + 7i)(3 - 2i)$
17. Simplify: $i + i^2 + i^3 + i^4$
18. Simplify: i^{43}
19. Divide: $\frac{4-5i}{3+2i}$

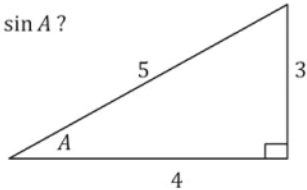
Miscellaneous

20. Evaluate: $\frac{8!}{4!2!}$

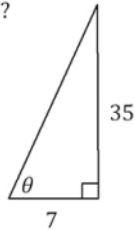
21. If $8! = k \cdot 6!$, then find k .
22. What is the vertex of $f(x) = (x + 3)^2 - 6$?
23. $\log_5 125 = ?$
24. Express $\log[x^3 \cdot \sqrt[4]{7}]$ as the sum or difference of logarithms.
25. If $A = \{0, 1, 2, 3\}$ and $B = \{3, 7, 11, 15\}$, then find the relationship of B to A .

Trigonometry

26. What is $\sin A$?

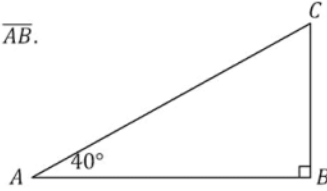


27. What equation defines θ ?

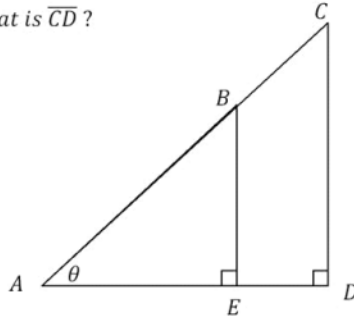


28. If $0^\circ < \theta < 90^\circ$ and $\cos \theta = \frac{2}{3}$, then find $\sin \theta$.
29. If $\pi < \theta < \frac{3\pi}{2}$ and $\sin \theta = \frac{-1}{9}$, then find $\cos \theta$.
30. If $\sin \theta = \frac{\sqrt{3}}{2}$, what is $\sin 300^\circ$?
31. If $\sin \theta = \frac{1}{2}$ where $0 \leq \theta \leq \frac{\pi}{2}$, what is $\sin 2\theta$?
32. If $\sin \theta = \frac{\sqrt{3}}{2}$ where $0 \leq \theta \leq \frac{\pi}{2}$, what is $\cos \frac{\theta}{2}$?
33. The graph of $y = \cos \theta$ is obtained by shifting the graph of $y = \sin \theta$ how many radians to the left?

34. If $\overline{CB} = 5$, then find \overline{AB} .



35. If $\overline{AB} = \overline{AD} = 1$, then what is \overline{CD} ?



36. If $y = 1 + \sin x$, what is y when $x = 3\pi$?

37. What is the radian measure of θ , where $0 \leq \theta \leq \pi$, if $\sqrt{\sin \theta} = 1$?

38. If $\cos^2 \theta + \cos \theta + \frac{1}{4} = 0$, where $0 < \theta < \pi$, then what is θ ?

Solutions

Functions

1. If $f(x) = x^2 - 3x + 2$ and $g(x) = 8x - 3$, then find $f[g(2)]$.

$$g(2) = 8(2) - 3 = 16 - 3 = 13$$

$$f(13) = 13^2 - 3(13) + 2 = 169 - 39 + 2 = \mathbf{132}$$

2. If $f(x) = x^2 - 3x + 5$, then find $f(a + 2)$.

$$f(a + 2) = (a + 2)^2 - 3(a + 2) + 5$$

$$= a^2 + 4a + 4 - 3a - 6 + 5$$

$$= \mathbf{a^2 + a + 3}$$

3. If $f(x) = 8x + 5$ and $g(x) = x^2 - 7$, then find $2f(3) - 4g(-2)$.

$$f(3) = 8(3) + 5 = 24 + 5 = 29 \quad g(-2) = -2^2 - 7 = 4 - 7 = -3$$

$$2f(3) - 4g(-2) = 2(29) - 4(-3) = 58 + 12 = \mathbf{70}$$

4. If $d = 1000 - 200q$ and $c = 4000 + 20d$, then find c as a function of q .

$$\begin{aligned} c &= 4000 + 20(1000 - 200q) \\ &= 4000 + 20000 - 4000q \\ &= \mathbf{24000 - 4000q} \end{aligned}$$

5. If $f(x) = x^2 - 3x + 2$ and $g(x) = 5 - x$, then find $f[g(x)]$.

$$\begin{aligned} f[g(x)] &= (5 - x)^2 - 3(5 - x) + 2 \\ &= 25 - 10x + x^2 - 15 + 3x + 2 \\ &= \mathbf{x^2 - 7x + 12} \end{aligned}$$

6. If $f(x) = 2x^2 + 1$ and $g(x) = 6x^3 - 2x^2 + 3x - 1$, then find $\frac{g(x)}{f(x)}$.

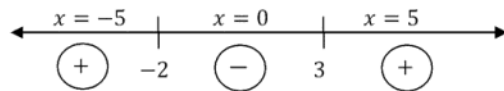
$$\frac{6x^3 - 2x^2 + 3x - 1}{2x^2 + 1} = \mathbf{3x - 1}$$

7. If the point $(5, -7)$ is on the graph of $y = f(x)$, what point must be on the graph of $y = f^{-1}(x)$?

The inverse of a function interchanges x and y . So, $(-7, 5)$ would be a point on the graph of $f^{-1}(x)$.

8. What is the domain of $f(x) = \sqrt{x^2 - x - 6}$?

$$x^2 - x - 6 \geq 0 \quad (x - 3)(x + 2) = 0 \quad x = 3 \quad x = -2 \quad (\infty, -2] \cup [3, \infty)$$



9. If $f(x) = \frac{2x+5}{3}$, then find $f^{-1}(x)$.

$$\begin{aligned} f(x) &= \frac{2x+5}{3} & y &= \frac{2x+5}{3} & \text{Interchange } x \text{ and } y. \\ x &= \frac{2y+5}{3} & 3x &= 2y+5 & \text{Solve for } y. \\ 3x-5 &= 2y & y &= \frac{3x-5}{2} & f^{-1}(x) = \frac{3x-5}{2} \end{aligned}$$

Exponents

10. If $4^x = 32$, then find x .

Make bases the same, then set exponents equal to each other.

$$4^x = 32 \quad (2^2)^x = 2^5 \quad 2x = 5 \quad x = \frac{5}{2}$$

11. Simplify $27^{-\frac{2}{3}}$

$$27^{-\frac{2}{3}} = \frac{1}{27^{\frac{2}{3}}} = \frac{1}{(\sqrt[3]{27})^2} = \frac{1}{3^2} = \frac{1}{9}$$

12. If $(\frac{2}{5})^{2x+4} = (\frac{5}{2})^{6x-5}$, then find x .

$$\left(\frac{2}{5}\right)^{2x+4} = \left(\frac{5}{2}\right)^{6x-5} \quad \text{Make denominators the same by flipping the fraction and changing the sign of the exponent.}$$

$$\left(\frac{2}{5}\right)^{2x+4} = \left(\frac{2}{5}\right)^{-6x+5} \quad 2x + 4 = -6x + 5 \quad 8x = 1 \quad x = \frac{1}{8}$$

13. If $(a^{x^2})\left(\frac{1}{a^{8x}}\right) = \frac{1}{a^{16}}$, then solve for x .

$$(a^{x^2})\left(\frac{1}{a^{8x}}\right) = \frac{1}{a^{16}} \quad (a^{x^2})(a^{-8x}) = a^{-16}$$

Make denominators the same by flipping the fraction thus changing the sign of its exponent.

$$x^2 - 8x = -16 \quad x^2 - 8x + 16 = 0 \quad (x - 4)(x - 4) = 0 \quad x = 4$$

14. If $(2^{x^2})\left(\frac{1}{2^{4x}}\right) = \frac{1}{16}$, then solve for x .

$$(2^{x^2})\left(\frac{1}{2^{4x}}\right) = \frac{1}{16} \quad (2^{x^2})(2^{-4x}) = 2^{-4} \quad \text{Change 16 to } 2^4 \text{ and move to the numerator.}$$

$$x^2 - 4x = -4 \quad x^2 - 4x + 4 = 0 \quad (x - 2)(x - 2) = 0 \quad x = 2$$

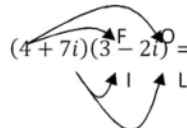
Complex Numbers

15. Multiply: $7i(4 - 3i)$

$$7i(4 - 3i) = 28i - 21i^2 = 28(i) - 21(-1) = \mathbf{21 + 28i}$$

16. Multiply: $(4 + 7i)(3 - 2i)$

FOIL - First, Outside, Inside, Last

$$(4 + 7i)(3 - 2i) = 12 - 8i + 21i - 14i^2 = 12 + 13i - 14i^2$$

$$= 12 + 13i - 14(-1) = 12 + 13i + 14 = \mathbf{26 + 13i}$$

17. Simplify: $i + i^2 + i^3 + i^4$

$$i + i^2 + i^3 + i^4 = i - 1 - i + 1 = \mathbf{0}$$

18. Simplify: i^{43}

Divide the exponent by 4 and match the remainder with the exponent.

$$4 \overline{) 43} \quad i^3 = -i$$

19. Divide: $\frac{4-5i}{3+2i}$

$$\frac{4-5i}{3+2i} \times \frac{3-2i}{3-2i} = \frac{12-8i-15i+10i^2}{9-6i+6i-4i^2} = \frac{12-8i-15i+10(-1)}{9-4(-1)} = \frac{12-23i-10}{9+4}$$
$$= \frac{2-23i}{13} = \frac{2}{13} - \frac{23i}{13}$$

Miscellaneous

20. Evaluate: $\frac{8!}{4!2!}$

$$\frac{8!}{4!2!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1} = \frac{1680}{2} = \mathbf{840}$$

21. If $8! = k \cdot 6!$, then find k .

$$8! = k \cdot 6!$$
$$k = 8 \cdot 7 = \mathbf{56}$$

22. What is the vertex of $f(x) = (x + 3)^2 - 6$?

For $f(x) = a(x - h)^2 + k$, the vertex is (h, k) , so the vertex of $f(x)$ is $(-3, -6)$

23. $\log_5 125 = ?$

$$\log_5 125 = \frac{\log 125}{\log 5} = 3$$

24. Express $\log[x^3 \cdot \sqrt[4]{7}]$ as separate log terms.

Each base gets its own log term and each exponent goes in front of that log term.

$$3 \log x + \frac{1}{4} \log 7$$

Terms in the numerator have a positive sign and any terms in the denominator have a negative sign in front of them.

25. If $A = \{0, 1, 2, 3\}$ and $B = \{3, 7, 11, 15\}$, then find the relationship of B to A .

$$B = \{3, \quad +4 \quad 7, \quad +4 \quad 11, \quad +4 \quad 15\}$$

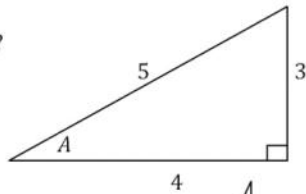
4x is part of the relationship because there is a difference of 4 between the numbers in Set B. Add or subtract the corresponding number in Set A to match.

$$B = 4A + 3$$

Trigonometry

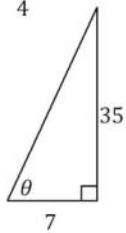
26. What is $\sin A$?

$$\sin a = \frac{3}{5}$$



27. What equation defines θ ?

$$\tan \theta = \frac{35}{7} = 5$$



28. If $0^\circ < \theta < 90^\circ$ and $\cos \theta = \frac{2}{3}$, then find $\sin \theta$.

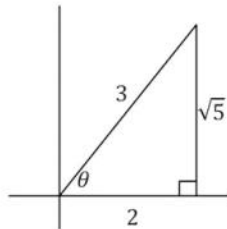
$$a^2 + b^2 = c^2$$

$$4 + b^2 = 9$$

$$b^2 = 5$$

$$b = \sqrt{5}$$

$$\sin \theta = \frac{\sqrt{5}}{3}$$

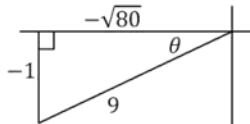


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29. If $\pi < \theta < \frac{3\pi}{2}$ and $\sin \theta = \frac{-1}{9}$, then find $\cos \theta$.

$$-\sqrt{81} = -4\sqrt{5}$$

$$\cos \theta = \frac{-4\sqrt{5}}{9}$$

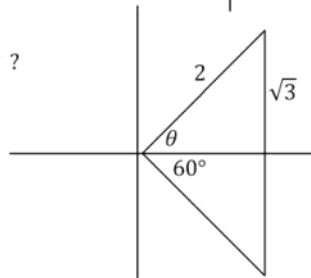


30. If $\sin \theta = \frac{\sqrt{3}}{2}$, what is $\sin 300^\circ$?

$$\theta = 60^\circ$$

$$300^\circ = 60^\circ \text{ in quadrant IV}$$

$$\sin 300^\circ = \frac{-\sqrt{3}}{2}$$

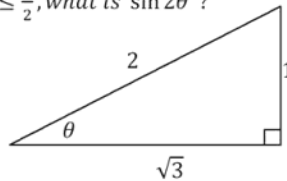


31. If $\sin \theta = \frac{1}{2}$ where $0 \leq \theta \leq \frac{\pi}{2}$, what is $\sin 2\theta$?

$$\theta = 30^\circ$$

$$2\theta = 60^\circ$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

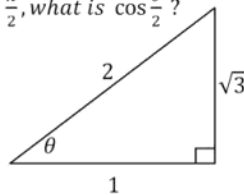


32. If $\sin \theta = \frac{\sqrt{3}}{2}$ where $0 \leq \theta \leq \frac{\pi}{2}$, what is $\cos \frac{\theta}{2}$?

$$\theta = 60^\circ$$

$$\frac{\theta}{2} = 30^\circ$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$



33. The graph of $y = \cos \theta$ is obtained by shifting the graph of $y = \sin \theta$ how many radians to the left?

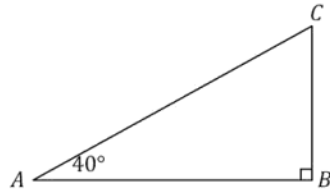
$$\frac{\pi}{2}$$

34. If $\overline{CB} = 5$, then find \overline{AB} .

$$\tan 40 = \frac{5}{\overline{AB}} \quad \overline{AB} = \frac{5}{\tan 40}$$

$$\tan 50 = \frac{\overline{AB}}{5} \quad \overline{AB} = 5 \tan 50$$

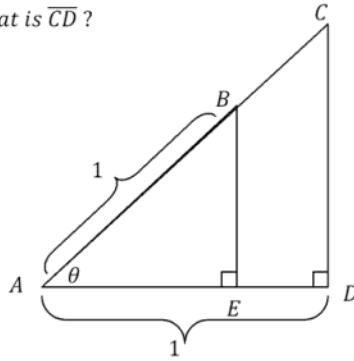
Either answer is correct.



35. If $\overline{AB} = \overline{AD} = 1$, then what is \overline{CD} ?

$$\tan \theta = \frac{\overline{CD}}{1}$$

$$\overline{CD} = \tan \theta$$



36. If $y = 1 + \sin x$, what is y when $x = 3\pi$?

$$y = 1 + \sin(3\pi) = \mathbf{1}$$

37. What is the radian measure of θ , where $0 \leq \theta \leq \pi$, if $\sqrt{\sin \theta} = 1$?

$$\sqrt{\sin \theta} = 1$$

$$\sin \theta = 1$$

$$\theta = \frac{\pi}{2}$$

38. If $\cos^2 \theta - \cos \theta + \frac{1}{4} = 0$, where $0 < \theta < \pi$, then what is θ ?

$$\left(\cos \theta - \frac{1}{2}\right)\left(\cos \theta - \frac{1}{2}\right) = 0$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = \cos^{-1} \frac{1}{2}$$

$$\theta = \mathbf{60^\circ}$$

